

**Bellringer #15**

13 12

The sum of two numbers is 25. The difference of the same two numbers is 1. What are the two numbers?

$$x + y = 25$$

$$x - y = 1$$

$$2x + \cancel{y} = 26$$

$$x = 13$$

$$y = 12$$

## **New Unit - Logic and Proofs**

**In this unit we will be discussing conditional statements and how to construct various types of proofs.**

Conditional Statement - a statement that can be written in if-then form.

**Short hand for if-then statements.**

$$p \longrightarrow q$$

*( read: If p, then q. )*

## LESSON 2-3 Conditional Statements

### Key Concept Conditional Statement

Words	Symbols
An <b>if-then statement</b> is of the form <u>if <math>p</math>, then <math>q</math>.</u>	$p \rightarrow q$ read <i>if <math>p</math> then <math>q</math>,</i> or <i><math>p</math> implies <math>q</math></i>
The <b>hypothesis</b> of a conditional statement is the phrase immediately following the word <u>if.</u>	$p$
The <b>conclusion</b> of a conditional statement is the phrase immediately following the word <u>then.</u>	$q$



**EXAMPLE 1**

Identify the Hypothesis and Conclusion

**B.** Identify the hypothesis and conclusion of the following statement.

Tamika will advance to the next level of play if she completes the maze in her computer game.

H

C



**EXAMPLE 2**

Write a Conditional in If-Then Form

**B.** Identify the hypothesis and conclusion of the following statement. Then write the statement in the if-then form.

A five-sided polygon is a pentagon.

↓  
H

C

If it's a five-sided polygon then it's a pentagon



**LESSON** **2–3** Conditional Statements



**EXAMPLE 2**

**A.** Which of the following is the correct if-then form of the given statement?

**A polygon with 8 sides is an octagon.**

- A.** If an octagon has 8 sides, then it is a polygon.
- B.** If a polygon has 8 sides, then it is an octagon.
- C.** If a polygon is an octagon, then it has 8 sides.
- D.** none of the above

**LESSON** **2–3** Conditional Statements



**EXAMPLE 2**

**B.** Which of the following is the correct if-then form of the given statement?

**An angle that measures  $45^\circ$  is an acute angle.**

**A.** If an angle is acute, then it measures less than  $90^\circ$ .

**B.** If an angle is not obtuse, then it is acute.

**C.** If an angle measures  $45^\circ$ , then it is an acute angle.

**D.** If an angle is acute, then it measures  $45^\circ$ .

## LESSON 2–3 Conditional Statements

### Truth Values

The hypothesis and conclusion of a conditional statement can have a truth value of true or false, as can the conditional statement itself. Consider the following example.

If Tom finishes his homework, then he will clean his room.

## 2–3 Conditional Statements

Hypothesis	Conclusion	Conditional	
Tom finishes his homework.	Tom cleans his room.	If Tom finishes his homework, then he will clean his room.	
T	T	T	If Tom <i>does</i> finish his homework and he <i>does</i> clean his room, then the conditional is true.
T	F	F	If Tom does <i>not</i> clean his room after he <i>does</i> finish his homework, then he has not fulfilled his promise and the conditional is false.
F	T	?	The conditional only indicates what will happen if Tom <i>does</i> finish his homework. He could clean his room or not clean his room if he does <i>not</i> finish his homework.
F	F	?	

## 2-3 Conditional Statements

Conditional Statements		
$p$	$q$	$p \rightarrow q$
T	T	T
<b>T</b>	<b>F</b>	<b>F</b>
F	T	T
F	F	T

Notice that a conditional is false *only* when its hypothesis is true and its conclusion is false.

Notice too that when a hypothesis is false, the conditional will *always* be considered true, regardless of whether the conclusion is true or false.

**EXAMPLE 3****Truth Values of Conditionals**

**A.** Determine the truth value of the conditional statement. If *false*, give a counterexample.

If you divide an integer by another integer, the result is also an integer.

C = false

H = True

False

$$\frac{2}{3} = 0.6\overline{6}$$

## 2-3 Conditional Statements

### EXAMPLE 3 Truth Values of Conditionals

**B.** Determine the truth value of the conditional statement. If *false*, give a counterexample.

If last month was February, then this month is March.

$H = \text{true}$

$C = \text{true}$

True

#### WatchOut!

##### Analyzing Conditionals

When analyzing a conditional, do not try to determine whether the argument makes sense. Instead, analyze the form of the argument to determine whether the conclusion follows logically from the hypothesis.



**EXAMPLE 3**

## Truth Values of Conditionals

**C.** Determine the truth value of the conditional statement. If *false*, give a counterexample.

When a rectangle has five sides, it is a parallelogram.

$H = \text{false}$

$C = \text{t or f}$

True



LESSON **2-3** Conditional Statements



**EXAMPLE 3**

**A.** Determine the truth value of the conditional statement. If *false*, give a counterexample.

The product of whole numbers is greater than or equal to 0.

$H = \text{true}$   
 $C = \text{true}$

- A. True**
- B. False**

LESSON **2-3** Conditional Statements



**EXAMPLE 3**

**B.** Determine the truth value of the conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

If yesterday was Tuesday, then today is Monday.

A. True

H = true

C = false

**B. False**

W follows T

LESSON **2-3** Conditional Statements



**EXAMPLE 3**

**C.** Determine the truth value of the conditional statement. If *false*, give a counterexample.

If a triangle has four sides, then it is concave.

$H = \text{False}$

$C = \text{True or False}$

**A. True**

**B. False**

### Key Concept Related Conditionals

Words	Symbols	Examples
A conditional statement is a statement that can be written in the form <i>if p, then q</i> .	$p \rightarrow q$	If $m\angle A$ is 35, then $\angle A$ is an acute angle.
The <b>converse</b> is formed by exchanging the hypothesis and conclusion of the conditional.	$q \rightarrow p$	If $\angle A$ is an acute angle, then $m\angle A$ is 35.
The <b>inverse</b> is formed by negating both the hypothesis and conclusion of the conditional.	$\sim p \rightarrow \sim q$	If $m\angle A$ is <i>not</i> 35, then $\angle A$ is <i>not</i> an acute angle.
The <b>contrapositive</b> is formed by negating both the hypothesis and the conclusion of the converse of the conditional.	$\sim q \rightarrow \sim p$	If $\angle A$ is <i>not</i> an acute angle then $m\angle A$ is <i>not</i> 35.

flip flop

not

not flip flop

Write the converse, inverse and contrapositive of the following.

Two angles that have the same measurement are congruent.

**Converse:**

**If two angles are congruent, then they have the same measurement**

**Inverse:**

**If two angles do not have the same measurement, then they are not congruent.**

**Contrapositive:**

**If two angles are not congruent, then they do not have the same measurement**

Assignment

Practice 2-3 all

